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Spontaneously Broken N=2 Supergravity Without Light Mirror Fermions

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ABSTRACT

We present a *spontaneously* broken N=2 supergravity model that reduces, in the flat limit $M_{Planck} \to \infty$, to a globally supersymmetric N=2 system with explicit soft supersymmetry breaking terms. These soft terms generate a mass $O(M_W)$ for mirror quarks and leptons, while leaving the physical fermions light, thereby overcoming one of the major obstacles towards the construction of a realistic N=2 model of elementary interactions.

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Introduction

In N=2 supersymmetric four-dimensional theories, all non-gravitational interactions are gauge interactions; therefore, N=2 is powerful enough to relate the Yukawa couplings to the gauge coupling(s), which are instead unrelated by N=1 supersymmetry. Exact N=2 supersymmetry also allows for the derivation of exact non-perturbative results on the dynamics of gauge theories [1]; some of these results survive the explicit breaking of N=2 by soft terms [2].

In spite of these attractive features, N=2 theories suffer from a serious problem that has hampered their use as realistic models of elementary interactions: all particles appear in *real* representations of the gauge group. Clearly, to recover the particle content of the standard model, in which particles belong to chiral (aka complex) representations of $SU(2) \times U(1)$, something has to happen.

Two mechanisms for generating chirality are known. The first requires a higher dimensional theory, which is itself chiral in higher dimensions. By compactifying to four dimensions, one finds that the mass term of fermions is given by the Dirac operator on the compact space. The number of chiral families (better, the mismatch between chiral and anti-chiral families) is then given by an index theorem. By wisely choosing the compactification, this index may be nonzero. A pioneering example of such compactification was given in [3]. The most successful example is the well known Calabi-Yau compactification of the heterotic string [4]. We must emphasize that these examples are four-dimensional theories with an *infinite* number of fields, all but a finite number of them have masses of the order of the inverse size of the compact dimensions. In other words, in these theories, N=2 supersymmetry is broken at the compactification scale, where the very notion of a four-dimensional space-time breaks down.

The second mechanism gives a very different scenario. There, the world is N=2 supersymmetric well below the compactification scale, or even the GUT scale; thus, the effective four-dimensional theory is N=2, with a *finite* number of fields. In this case, one can still recover the particle content of the standard model by giving Majorana masses to the "mirror" fermions belonging to the wrong-chirality representation of $SU(2) \times U(1)$. Since Majorana masses necessarily break the gauge group, one finds a model-independent constraint on the mirror masses: they must be of the order of the $SU(2) \times U(1)$ breaking:

$$M_{mirror} \sim 100\, GeV. \tag{1}$$

The existence of mirror fermions is still compatible with experiment, for appropriate mixing angles with physical fermions [5]. This model-independent constraint, at least, makes N=2 models interesting, since they make a definite prediction that can be verified by future experiments.

An N=2 model which implements this scenario was proposed in [6]. Two difficulties face anyone attempting to generate tree-level mirror-fermion masses in N=2 supersymmetry.

The first, solved in ref. [6], is that physical fermions, belonging to hyper-multiplets [7], can only get tree-level masses by the VEV of a complex scalar, supersymmetric partner of the gauge field, and belonging to the adjoint representation of the gauge group. This adjoint field must play the role of the standard-model Higgs field. This fact requires an extension of both the standard-model gauge group and the Higgs sector. The minimal such extension, given in [6], is as follows.

- The gauge group is extended to $SU(3) \times SU(4) \times U(1)$.
- $SU(4) \times U(1)$ is broken to $SU(2) \times U(1)$ by two Higgs hypermultiplets in the $(1, 4, +1/2) \oplus (1, \bar{4}, -1/2)$ and $(1, 4, -1/2) \oplus (1, \bar{4}, +1/2)$ of the gauge group, respectively.
- Quarks and leptons, together with their mirrors, belong to real representations of the gauge group. In the notations of ref. [6]:

$$X_L = \begin{pmatrix} L \\ \bar{L}' \end{pmatrix} \sim (1, 4, -1/2), \qquad Y_L = \begin{pmatrix} L' \\ \bar{L} \end{pmatrix} \sim (1, \bar{4}, +1/2),$$
$$X_Q = \begin{pmatrix} Q \\ \bar{Q}' \end{pmatrix} \sim (3, 4, +1/6), \qquad Y_Q = \begin{pmatrix} Q' \\ \bar{Q} \end{pmatrix} \sim (\bar{3}, \bar{4}, -1/6). \tag{2}$$

  Here Q, L denote the physical quarks and leptons, while Q', L' denote the mirrors. Notice that a given irreducible representation of the gauge group contains both physical fermions and mirrors.

The second difficulty proved harder to solve. In order to achieve the right pattern of symmetry breaking, the authors of [6] introduce by hand some soft terms, which preserve the good ultraviolet properties of the N=2 theories [8, 9], but that, on the other hand, *explicitly* break the N=2 supersymmetry. Ref. [6] suggests that these terms may come from a *spontaneously* broken N=2 supergravity, much in the same way as the corresponding terms arise in N=1 supergravity (see [10] and references therein).

The quest for such a spontaneously broken N=2 supergravity has been elusive so far. Indeed, even though N=2 supergravity models spontaneously broken to N=1 exist [11, 12, 13], none has been found, which generates mirror-fermion masses.

Purpose of this paper is to exhibit such a supergravity model. This model has the field content of ref [6], supplemented with the minimal hidden sector necessary to *spontaneously* break N=2 supersymmetry with two independent scales. In the flat limit where the Planck mass $M_P \to \infty$, while the mass of both gravitini is kept constant, the SUSY breaking generates soft terms (tri-linear Yukawa couplings and masses), which allow one to recover the model of ref. [6] in an appropriate phenomenologically realistic range of parameters. This model removes

the major (though not unique) obstacle to the construction of realistic models where N=2 supersymmetry is spontaneously broken well below the Planck (or compactification) scale.

This paper is organized as follows: in Section 1 we review the basic facts about supergravity Lagrangians, using the geometric formulation of [14]; in Section 2 we review the construction of the softly broken N=2 model with tree-level mirror splitting of ref. [6]. Section 3 describes how to spontaneously break N=2 supergravity with two independent scales. Section 4 is the heart of the paper. There, we show how to recover the soft supersymmetry breaking terms needed to give mass to the mirror fermions from the flat limit of an N=2 supergravity. Some concluding remarks are given in Section 5, while Appendix A contains an explicit construction of the hypermultiplet manifold needed in Section 4.

The reader interested in the results, rather than in details of the construction can skip Sections 1 and 3.

1 N=2 Supergravity Lagrangians

The fields of N=2 supergravity belong to the graviton multiplet, the vector multiplet and the hypermultiplet. The graviton multiplet contains the graviton, two gravitinos and a vector field. The vector multiplet contains a vector field, a complex scalar and a Majorana fermion. The hypermultiplet contains four real scalars and a Dirac fermion.

The bosonic part of the N=2 vector multiplet contains a complex scalar z^i in addition to the gauge field; supersymmetry constrains the scalar to parametrise a special Kähler manifold of real dimension $2n_V$ (where n_V is the number of vector multiplets in the theory). The special geometry of the vector-multiplet manifold is specified by $2(n_V+1)$ holomorphic functions $X^\Sigma(z^i), F_\Sigma(z^i)$ $(i=1,..,n_V, \Sigma=0,..,n_V)$ [15] , in terms of which the Kähler potential reads:

$$K = -\log i(X^{*\Lambda}F_\Lambda - X^\Lambda F^*_\Lambda). \tag{3}$$

The metric on the scalar manifold is $g_{ij^*} = \partial_i\partial_{j^*}K$. In N=2 supergravity there is an extra vector field which belongs to the graviton multiplet, called graviphoton; no scalar fields is associated to it. Roughly speaking, the index Λ labels all vector fields (n_V+1), while the index i labels the complex scalars (n_V) or, equivalently, the vector multiplets.

In a coordinate-independent definition of a special Kähler manifold, the functions (X^Λ, F_Λ) are holomorphic sections of a $2n_V+2$ dimensional symplectic bundle over the manifold. The notion of a prepotential $F(X)$ which characterizes the N=2 literature in the context of supergravity [16, 15] as well as in the rigid limit [1], can be recover only in particular cases and it is associated with the choice of "special" coordinates. The prepotential exists provided that the matrix

$$e^a_i(z) = \partial_i\left(\frac{X^a}{X^0}\right) \tag{4}$$

is invertible. In this case X^Λ can be regarded as a set of homogeneous coordinates for the special Kähler manifold, and

$$F_\Lambda(X) = \frac{\partial}{\partial X^\Lambda} F(X), \tag{5}$$

where the $F(X)$ is a homogeneous function of degree 2. Under these circumstances one can use *special coordinates* $t^a = X^a/X^0$, and the whole geometry is encoded in a single holomorphic prepotential $(X^0)^{-2}F(X)$.

As shown in Section 4, we find a pattern of supersymmetry breaking which splits the mirrors by using a choice of sections for which the prepotential does not exist; such sections can be obtained, for example, by applying an appropriate symplectic transformation to sections derived from a prepotential.

N=2 hypermultiplets contain four real scalars. N=2 supergravity requires such scalars to be the coordinates of a quaternionic manifold [17].

A quaternionic manifold is a $4n_H$-dimensional real manifold with three complex structures J^x that satisfy the quaternionic algebra

$$J^x J^y = -\delta_{xy} + \epsilon^{xyz} J^z, \quad x = 1, 2, 3, \tag{6}$$

such that the metric $ds^2 = h_{uv} dq^u dq^v, u, v = 1, .., 4n_H$ is hermitian with respect to them. The two-forms $K^x_{uv} = h_{uw}(J^x)^w_v$ are covariantly closed with respect to an $SU(2)$ connection ω^x

$$\nabla K^x \equiv dK^x + \epsilon^{xyz} \omega^y \wedge K^z = 0. \tag{7}$$

To complete the definition of a quaternionic manifold we must impose that the curvature Ω^x of the connection ω^x is proportional to the form K^x

$$\Omega^x \equiv d\omega^x + \frac{1}{2}\epsilon^{xyz}\omega^y \wedge \omega^z = \lambda K^x. \tag{8}$$

The proportionality coefficient between K and Ω is arbitrary; the choice $\lambda = -M_P^{-2}$ gives the correct normalization of the kinetic terms in the supergravity Lagrangian. Notice that the definition of a hyperkähler manifold differs only in that Ω^x vanishes, rather than being proportional to K^x. This corresponds to taking the limit $M_P \to \infty$ in eq. (8).

The gauge group is a subgroup of the isometries of the total scalar manifold parametrised by z^i and q^u. Since there are $n_V + 1$ vectors, the gauge group has dimension $n_V + 1$; its action on the scalars is given by $n_V + 1$ Killing vectors:

$$\begin{aligned} z^i &\to z^i + \epsilon^\Lambda k^i_\Lambda(z), \\ q^u &\to q^u + \epsilon^\Lambda k^u_\Lambda(q) \ . \end{aligned} \tag{9}$$

To write the N=2 Lagrangian, one must introduce a triplet of real prepotential for the quaternionic manifold [16, 15]. They are the supergravity generalization of the triplet of D-terms of

rigid N=2 supersymmetric gauge theories, and reduce to them in the flat limit. They are defined as the three real functions P^x_Λ which satisfy

$$2k^u_\Lambda \Omega^x_{uv} = -\nabla_v P^x_\Lambda = -(\partial_v P^x_\Lambda + \epsilon^{xyz}\omega^y_v P^z_\Lambda). \tag{10}$$

Now we have all the ingredients to write the (bosonic part of) the N=2 supergravity Lagrangian [15]:

$$\begin{aligned}\mathcal{L}_{bos} &= -\frac{1}{2}R + g_{ij^*}\nabla^\mu z^i \nabla_\mu \bar{z}^{j^*} + h_{uv}\nabla_\mu q^u \nabla^\mu q^v + \\ &\quad \mathrm{i}\left(\bar{\mathcal{N}}_{\Lambda\Sigma}\mathcal{F}^{-\Lambda}_{\mu\nu}\mathcal{F}^{-\Sigma\mu\nu} - \mathcal{N}_{\Lambda\Sigma}\mathcal{F}^{+\Lambda}_{\mu\nu}\mathcal{F}^{+\Sigma\mu\nu}\right) - V(z,\bar{z},q),\end{aligned} \tag{11}$$

where $\mathcal{F}^{\pm\Lambda}_{\mu\nu} = \frac{1}{2}(\mathcal{F}^\Lambda_{\mu\nu} \pm \frac{\mathrm{i}}{2}\epsilon^{\mu\nu\rho\sigma}\mathcal{F}^\Lambda_{\rho\sigma})$. If we define $(f^\Lambda_i, h_{i\Lambda}) = (\partial_i + \partial_i K)(X^\Lambda, F_\Lambda)$, the matrix $\mathcal{N}_{\Lambda\Sigma}$ is determined by

$$F_\Lambda = \mathcal{N}_{\Lambda\Sigma}X^\Sigma, h_{i^*\Lambda} = \mathcal{N}_{\Lambda\Sigma}f^\Sigma_{i^*}. \tag{12}$$

$\mathcal{N}$ is the scalar-dependent gauge kinetic term. When a prepotential exists, it reduces to the familiar [16, 1] expression $\frac{\partial^2 F}{\partial X^\Lambda \partial X^\Sigma}$. The potential is given by:

$$V(z,\bar{z},q) = e^K\left[\left(g_{ij^*}k^i_\Lambda k^{j^*}_\Sigma + 4h_{uv}k^u_\Lambda k^v_\Sigma\right)\bar{X}^\Lambda X^\Sigma + \left(g^{ij^*}f^\Lambda_i f^\Sigma_{j^*} - 3\bar{X}^\Lambda X^\Sigma\right)\mathcal{P}^x_\Lambda \mathcal{P}^x_\Sigma\right]. \tag{13}$$

Since we are interested in the flat limit $M_P \to \infty$, we must restore physical normalizations in the previous Lagrangian, in which all the fields are dimensionless. The correct assignment is to restore the right dimension for the Lagrangian by multiplying it by M_P^4 and to keep dimensionless the fields in the graviton multiplets and in the hidden sector of the theory, while restoring dimensions in the generic field x in the physical sector by writing it as x/M_P. The flat limit is then obtained by sending $M_P \to \infty$ while keeping x finite. The hidden sector will trigger supersymmetry breaking and interference terms between the hidden and physical sectors will give rise to soft supersymmetry breaking terms in the flat limit. These terms are exhaustively discussed in Section 4. For the time being, let us focus on the physical sector and let us work out the simplifications in the previous formulas due to the flat limit.

Let us denote with z^i, b^u, respectively, the scalar partners of the gauge fields and the hypermultiplets scalars in the hidden sector, and denote with ϕ^i, q^u the same quantities in the physical sector. Since the kinetic term for the hidden-sector scalars is proportional to M_P^2, in the flat limit, all hidden-sector scalars "freeze" to their vacuum expectation value. Also, when $M_P \to \infty$, all the previous quantities can be expanded in powers of ϕ^i, q^u. A standard dimensional argument says that in the physical sector only renormalizable terms will survive in this limit.

By expanding the Kähler potential and the quaternionic metric in inverse powers of M_P, and keeping only the leading order, all the scalar metrics become obviously flat. By appropriately

choosing the supergravity metric, all observable-sector fields will be canonically normalized in the flat limit.

First, Let us describe the hypermultiplets. The metric is flat and can be normalized to $h_{uv} = \delta_{uv}$. The quaternionic geometry reduces to the hyperkähler one in the flat limit, since Ω^x is proportional to $1/M_P^2$.

It is convenient to represent the four scalars q^u in a hypermultiplet as a quaternion

$$\begin{aligned} Q &= e_u q^u = \begin{pmatrix} x & -y^* \\ y & x^* \end{pmatrix}, \\ q^u &= \frac{1}{2}\mathrm{tr}\,\bar{e}^u Q, \end{aligned} \tag{14}$$

where $e_u = (1, -i\vec{\sigma}), \bar{e}^u = (1, i\vec{\sigma})$.

There is an alternative representation of the hypermultiplets in the N=2 literature [16] in which the scalar fields A_i^a have an index i, denoting that it is a doublet of the $SU(2)$ global R-symmetry, and an extension index which transforms under the gauge group. They must satisfy the reality constraint:

$$(A_i^a)^* = \epsilon^{ij}\rho_{ab}A_j^b, \tag{15}$$

where $\rho\rho^* = -1$ for consistence. This can be solved only if the space labeled by a is even-dimensional. A convenient choice for ρ is a block diagonal form in which the entries are $-i\sigma_2$. For a single hypermultiplet the solution of the reality constraint is exactly the quaternion in eq. (14).

The linear action of the gauge group,

$$\delta^\Lambda Q = -i\mathcal{T}^\Lambda Q, \tag{16}$$

is constrained by the reality condition to:

$$\mathcal{T}^\Lambda = \begin{pmatrix} T^\Lambda & 0 \\ 0 & -(T^\Lambda)^* \end{pmatrix}, \tag{17}$$

where T^Λ is a hermitian generator of the gauge group. This equation implies that, for example, if x transforms in the fundamental representation of the gauge group, y transforms in the anti-fundamental. The Killing vectors are:

$$k_\Lambda^u = -\frac{i}{2}\mathrm{tr}\,\bar{e}^u\mathcal{T}_\Lambda Q. \tag{18}$$

The three complex structures are

$$(J^x)_v^u = \frac{-i}{2}\mathrm{tr}\,\bar{e}^u\sigma^x e_v. \tag{19}$$

They correspond to the left multiplication of the quaternion by $-i\vec{\sigma}$ and therefore satisfy the quaternionic algebra.

In the flat limit, the covariant $SU(2)$ derivative simplifies to the ordinary differential in the equation for the prepotential:

$$2k_\Lambda^u h_{uw} \vec{J}_v^w = \partial_v \vec{P}_\Lambda, \tag{20}$$

which can be solved to give

$$\vec{P}_\Lambda = \frac{1}{2} \mathrm{tr}\, \vec{\sigma} Q^\dagger \mathcal{T}_\Lambda Q. \tag{21}$$

Notice that for an Abelian gauge field we have:

$$\mathcal{T} = q\sigma_3. \tag{22}$$

It will be sometimes useful to define the vector

$$\mathcal{Q} = \begin{pmatrix} x \\ y \end{pmatrix}. \tag{23}$$

As for the gauge-field Lagrangian, we choose for the physical fields the sections

$$(X^a, F_a) = \left(\frac{1}{\sqrt{2}} g\phi^a, \frac{-i}{\sqrt{2}g} g_{ab}\phi^b \right), \tag{24}$$

where $g_{ab} = \mathrm{tr}\,(T_a T_b)$ and define $\phi = \phi^a T_a$. Expanding the Kähler potential, the bosonic part of the Lagrangian (for a single factor in the gauge group and only one hypermultiplet in the fundamental) reduces to

$$\mathcal{L} = -\frac{1}{g^2} \mathrm{tr}\, F_{\mu\nu} F^{\mu\nu} + \mathrm{tr}\, \nabla\phi\nabla\phi^\dagger + \frac{1}{2} \mathrm{tr}\, \nabla\mathcal{Q}^\dagger \nabla\mathcal{Q} - V(\phi, \mathcal{Q}), \tag{25}$$

with the potential:

$$V(\phi, \mathcal{Q}) = g^2 \left(\mathrm{tr}\, \left(\left[\phi, \phi^\dagger \right] \right)^2 + \mathrm{tr}\, \mathcal{Q}^\dagger \left\{ \phi, \phi^\dagger \right\} \mathcal{Q} + \vec{P}_a \vec{P}_b (g^{-1})_{ab} \right). \tag{26}$$

Any dependence on the frozen moduli z^i has been re-absorbed in the normalization for ϕ. One can recognize the standard rigid N=2 gauge Lagrangian. The triplet of prepotentials has reduced to the triplets of D-terms of N=2 supersymmetry.

2 Soft Breaking Terms and Mirror Splitting

A concrete example of softly broken N=2 (rigid) supersymmetry with tree-level mirror fermion mass splitting was given in [6]; in this section, we review that example. The model in [6] has a gauge group $SU(3) \times SU(4) \times U(1)$, and the physical quarks and leptons are arranged in representations of the gauge group as in eq. (2). The Higgs sector responsible for the breaking

of $SU(4)\times U(1)$ is made of a scalar field, ϕ, partner of the $SU(4)\times U(1)$ gauge field under N=2 supersymmetry, and four complex scalars (x_i, y_i), $i=1,2$, arranged in two hypermultiplets. The potential simplifies if we include the coupling constants dependence in Φ

$$\Phi_i = g_4 T_i^I \phi_I + g_1 Z_i \phi, \tag{27}$$

where T_i^I are the generators of $SU(4)$ and Z_i is the generator of $U(1)$. This definition will be used only in the minimization of the potential. Obviously, in the kinetic term the fields $T_i^I\phi_I$, ϕ appear separately. The normalizations are as in formula (25). The index $i=1,2$ in the definition above labels the different representations of the gauge group acted upon by Φ; we shall omit it wherever unnecessary. The first hypermultiplet transforms in the $(1,4,+1/2)\oplus(1,\bar{4},-1/2)$ of the gauge group, while the second transforms in the $(1,4,-1/2)\oplus(1,\bar{4},+1/2)$. The leptons and quarks (physical and mirror) get their tree-level masses only from the term

$$\overline{X}_L \Phi Y_L, \quad \overline{X}_Q \Phi Y_Q. \tag{28}$$

In order to give a large mass to the mirrors, while keeping the physical fermions light, the VEV of Φ must be off-diagonal. This never happens with a pure N=2 potential.

Indeed, the N=2 supersymmetric scalar potential depends on these fields as follows:

$$\begin{aligned}
V_{N=2}(x_i, y_i, \Phi) \quad = \quad & \Big\{\frac{1}{g_4^2}\mathrm{tr}\,([\Phi,\Phi^\dagger])^2 + \sum_i (x^\dagger\{\Phi_i^\dagger,\Phi_i\}x_i + y_i^\dagger\{\Phi_i^\dagger,\Phi_i\}y_i)\Big\} + \\
& g_4^2\Big\{\frac{1}{4}\sum_{ij}|x_i^\dagger x_j + y_i^\dagger y_j|^2 + \frac{1}{2}\sum_{ij}\{(y_j^\dagger y_i)(x_j^\dagger x_i) - (y_j^t x_i)(x_i^\dagger y_j^*)\} + \\
& -\frac{1}{16}(\sum_i |x_i|^2 + \sum_i |y_i|^2)^2 + \frac{1}{4}(\sum_i |x_i|^2)(\sum_j |x_j|^2) + \\
& -(\sum_i y_i^t x_i)(\sum_j x_j^\dagger y_j^*)\Big\} + \frac{1}{8}\Big\{[\sum_i q_i(|x_i|^2 - |y_i|^2)]^2 + \\
& 4\sum_{ij} q_i q_j (y_i^t x_i)(x_j^\dagger y_j^*)\Big\}.
\end{aligned} \tag{29}$$

Here $q_1=1$, $q_2=-1$, and normalizations are as in formula (25) for each factor in the gauge group.

In the absence of soft breaking terms, the potential is always non-negative, and it has a global minimum ($V_{N=2}=0$) at

$$x_i = y_i = 0, \quad [\Phi^\dagger, \Phi] = 0. \tag{30}$$

The vanishing of the commutator implies that Φ is diagonal up to a gauge rotation.

An off-diagonal VEV for Φ can be obtained by introducing appropriate soft terms (scalar masses and scalar tri-linear couplings) which explicitly break the rigid N=2 supersymmetry,

but preserve the most important property of supersymmetry, namely, the absence of quadratic divergences [8]. These terms are:

$$V_{soft} = \sum_i \Big[m_{x_i}^2 |x_i|^2 + m_{y_i}^2 |y_i|^2 + m_{\Phi_i}^2 |\Phi_i|^2 + \Big(M_i y_i^t \Phi_i x_i + c.c. \Big) \Big]. \tag{31}$$

The mass parameters m_{x_i}, m_{y_i},m_{Φ_i}, and M_i (which are complex, in general) can be adjusted to give VEVs of arbitrary magnitude to x_i, y_i, and Φ. The minimization of the potential $V_{N=2} + V_{soft}$ is arduous, for arbitrary values of $V_i \equiv |\langle x_i \rangle|$, $v_i \equiv |\langle y_i \rangle|$, and $g_4 \hat{v} \equiv |\langle \Phi \rangle|$, but it becomes doable in the approximation $V_i \gg \hat{v} \gg v_i$ [6].

In this limit, there is only one term $O(V_i^4)$ in the potential which contributes to the alignment of x_i: $1/2|x_1^\dagger x_2|^2$. It favors $x_i \perp x_2$, so that we can choose

$$\langle x_1 \rangle = \begin{pmatrix} 0 \\ 0 \\ V_1 \\ 0 \end{pmatrix}, \quad \langle x_2 \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ V_2 \end{pmatrix}. \tag{32}$$

The first VEV breaks $SU(4) \times U(1)$ to $SU(2)_L \times SU(2)_R \times U(1)$ while the second breaks this group to $SU(2) \times U(1)$.

The next largest terms which contribute to alignment are $O(V_i^2 \hat{v}^2)$, and read:

$$\sum_i \Big[x_i^\dagger \{\Phi_i^\dagger, \Phi_i\} x_i + \Big(M_i y_i^t \Phi_i x_i + c.c. \Big) \Big]. \tag{33}$$

By consistency, $M_i = O(g_4 V_i \hat{v}/v_i) \gg V_i$. The minimization of eq. (33) with respect to Φ can be done exactly since Φ appears there quadratically, and gives

$$\begin{aligned} \Phi_{a3} &= -\frac{M_1^*}{V^*} y_{1\,a}^*, & \Phi_{a4} &= -\frac{M_2^*}{V^*} y_{2\,a}^*, \\ \Phi_{43} &= -\frac{M_1^*}{2V^*} y_{1\,4}^*, & \Phi_{34} &= -\frac{M_2^*}{2V^*} y_{2\,3}^*, \\ \Phi_{1\,33} &= -\frac{M_1^*}{2V^*} y_{1\,3}^*, & \Phi_{2\,44} &= -\frac{M_2^*}{2V^*} y_{2\,4}^*, \\ \Phi_{3a} &= \Phi_{4a} = 0, & & a, b = 1, 2. \end{aligned} \tag{34}$$

Here, for simplicity, and without loss of generality, we assumed $V_1 = V_2 \equiv V$. The potential term in eq. (33), computed at the stationary point for Φ given in eq. (34), gives rise to a *negative* mass term for y_i:

$$-\sum_i \left(\frac{1}{2} |M_i y_{i\,3}|^2 + \frac{1}{2} |M_i y_{i\,4}|^2 \right) - \sum_{ai} M_i^2 |y_{i\,a}|^2. \tag{35}$$

For an appropriate *positive* value of the mass term $m_{y_i}^2$ in V_{soft}, the minimum in y_i becomes

$$y_{i\,3} = y_{i\,4} = 0, \quad y_{i\,a} \neq 0. \tag{36}$$

Finally, the only $O(\hat{v}^4)$ alignment term in the potential, tr $[\Phi^\dagger\Phi]^2$, implies that, in the presence of small, positive mass terms $m^2_{\Phi_i}$, the potential $V_{N=2}+V_{soft}$ is minimized by an off-diagonal VEV of Φ, which, by an $SU(2)_L$ rotation, can be brought into the form

$$\langle\Phi\rangle = g_4 \begin{pmatrix} 0 & 0 & \hat{v}_1 & 0 \\ 0 & 0 & 0 & \hat{v}_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{37}$$

The off-diagonal elements of Φ play here the role of the standard-model Higgs, and their VEVs $\hat{v}_i$ break $SU(2)\times U(1)$ to $U(1)$. The electric charge is $Q = T_{3L} + T_{3R} + Z$, where T_{3L}, T_{3R} are the diagonal generators of $SU(2)_L$, $SU(2)_R$, and Z is $1/2(B-L)$ on matter fields.

The Φ in eq. (37) generates a mass $M_U = \sqrt{2}\hat{v}_1 g_4$ for the mirror up quarks and leptons, while the mirror down quarks and leptons have a mass $M_D\sqrt{2}\hat{v}_2 g_4$. The constraint on the mirror masses eq. (1) now becomes, more precisely:

$$M_U^2 + M_D^2 = 2M_W^2. \tag{38}$$

Additional sub-dominant terms in the potential ($O(v^2\hat{v}^2)$ etc.) may generate small nonzero VEVs for the block-diagonal components of Φ. These terms induce small mixing between mirror and physical fermions. These mixings are not in contradiction (indeed, they are favored) by experimental data [5].

3 The Supersymmetry Breaking Mechanism

The simplest method to break rigid supersymmetry is to introduce Fayet-Iliopoulos (FI) terms for a $U(1)$ gauge fields in the theory. In an N=2 gauge theory one can introduce three FI terms corresponding to constant pieces in the triplet of D-terms. The D-terms appear in the supersymmetric transformation formula for the gaugino in a combination which, roughly speaking, is $\delta\lambda = \vec{D}\vec{\sigma}\epsilon$. A nonzero expectation value for $\vec{D}$ breaks supersymmetry.

In the local case the D-terms are replaced by the prepotentials $\vec{P}^\Lambda$, which appear in the gaugino and gravitino shifts. For example, the gravitino shift reads:

$$\delta\psi_{A\,\mu} = -\frac{1}{2}e^{K/2}(\vec{\sigma})_A^{\ C}\epsilon_{BC}\vec{P}_\Lambda X^\Lambda(z)\gamma_\mu\eta^B \equiv iS_{AB}\gamma_\mu\eta^B. \tag{39}$$

A nonzero value for $\vec{P}_\Lambda X^\Lambda(z)$ breaks supersymmetry. We choose prepotentials and sections in such a way that $\vec{P}_\Lambda X^\Lambda(z)$ has an (essentially) constant complex piece. Let us remind the reader that we need to break supersymmetry with two different arbitrary scale (two different masses for the two gravitinos), in order to break the global $SU(2)$ R-symmetry. This is required because we want to give different masses for the scalars x_i and y_i of Section 2, and they are doublets

under the R-symmetry. The gravitino mass matrix is S_{AB}; by a suitable choice of the complex number $\vec{P}_\Lambda X^\Lambda(z)$ it can have two arbitrary eigenvalues.

A minimal supergravity model with all these characteristics was constructed in [12], where it was used to partially break N=2 supersymmetry to N=1 in Minkowsky space. In this model, the charged-hypermultiplet scalars parametrise the quaternionic manifold $SO(4,1)/SO(4)$, and the vector-multiplet scalars parametrise the Kähler manifold $SU(1,1)/U(1)$.

Let us denote the quaternionic coordinates of the hypermultiplet manifold by b^u, $u = 0,1,2,3$. The metric is $h_{uv} = (1/2b^{0 2})\delta_{uv}$ and the three complex structures are exactly as in eq. (20). The quaternionic potentials read:

$$\omega^x_u = \frac{1}{b^0}\delta^x_u, \quad \Omega^x_{0u} = -\frac{1}{2b^{0 2}}\delta^x_u, \quad \Omega^x_{yz} = \frac{1}{2b^{0 2}}\epsilon^{xyz}, \quad x,y,z = 1,2,3. \tag{40}$$

The manifold is invariant under arbitrary constant translation of the coordinates b^1, b^2, b^3. We can then choose constant Killing vectors $k^u_\Lambda = \delta_{ux}\zeta^x_\Lambda$, where ζ^x_Λ are arbitrary constants.

The prepotentials can be determined using the formulas of Section 1, and they read:

$$P^x_\Lambda = \frac{1}{b_0}\zeta^x_\Lambda. \tag{41}$$

The $\vec{P}_\Lambda$ are real functions, nonzero and independent of the vector multiplets. Excluding the dependence on b_0, which becomes irrelevant in the flat limit, they can be considered as three real constants, exactly as FI terms are expected to be.

An undesired dependence on z in $\vec{P}_\Lambda X^\Lambda(z)$ can be avoided using the sections

$$X^0(z) = -\frac{1}{\sqrt{2}}, \quad X^1(z) = \frac{i}{\sqrt{2}}, \quad F_0 = \frac{i}{\sqrt{2}}z, \quad F_1 = \frac{1}{\sqrt{2}}z. \tag{42}$$

This choice gives the $SU(1,1)/U(1)$ Kähler potential

$$K = -\log(z+\bar{z}). \tag{43}$$

Note that no prepotential exists for such a choice of sections [4]. The absence of prepotential is necessary in order to get a partial breaking to N=1 at zero cosmological constant. We will make the same choice of sections, since it simplifies the formulas and avoids a z dependence in the sections.

With an appropriate choice of the complex vector

$$\vec{\mathcal{M}} = X^0\vec{P}_0 + X^1\vec{P}_1 = \frac{1}{\sqrt{2}}(\vec{\zeta}_0 + i\vec{\zeta}_1), \tag{44}$$

the gravitino mass matrix, $\vec{\sigma}\vec{\mathcal{M}}\sigma_2$, can be given two arbitrary eigenvalues. In this way we can break supersymmetry to N=0 with two arbitrary scales.

[4] One can find these sections by the symplectic transformation (electric-magnetic duality) $X^1 \to -F_1$, $F_1 \to X^1$ of the basis specified by the prepotential $F(X^\Lambda) = iX^0X^1$.

4 The Supergravity Model

In this section we construct a supergravity model which, in the flat limit, reproduces the soft breaking terms discussed in Section 2 and lifts the mass degeneracy between mirror and physical fermions. We work in the approximation in which supergravity formulas reduce to those of rigid supersymmetry in the physical sector, and the kinetic term of the physical fields is canonical. In other words, we keep only the first term of the supergravity Lagrangian in the $1/M_P$ expansion.

Let us start the description of our model.

Let us begin with the quaternionic manifold of the hypermultiplets. In the full supergravity model at the scale M_P the hidden and physical hypermultiplets parametrise a complicated quaternionic manifold, which in the flat limit reduces to the product of the hidden-sector quaternionic manifold $(SO(4,1)/SO(4))$ times quaternions with flat metric. The question of whether this quaternionic manifold exists is answered in Appendix A, where such space is explicity constructed.

The multiplets of our model are those of the rigid N=2 theory described in Section 2, coupled through gravitational interaction to the hidden sector described in Section 3. The only thing we need to remember about the hidden sector is that the quaternionic manifold $SO(4,1)/SO(4)$ admits three independent "translations" as isometries, which can be used to break supersymmetry by giving a (complex) constant term to the quantity $\vec{P}_\Lambda X^\Lambda(z)$. To these fields, we add a hypermultiplet $(\tilde{x}, \tilde{y})$ with a negative kinetic term, neutral under the physical gauge group and a non-propagating, auxiliary, Abelian vector multiplet. The two Higgs hypermultiplets (x_i, y_i) have charges q_i under the $U(1)$ vector of this multiplet while $(\tilde{x}, \tilde{y})$ has charge p, and the scalars in $SO(4,1)/SO(4)$ translate by the constant vector $\vec{h}$. The introduction of non-propagating gauge fields in N=2 supergravity is also known as quaternionic quotient [18] and it is one of the most powerful ways to construct new quaternionic manifolds from known ones. Since this auxiliary gauge field has no kinetic term, it can be eliminated, together with its supersymmetric partners, using their equation of motion. In particular, $\vec{P}_{aux} = 0$, allows to express $(\tilde{x}, \tilde{y})$ in terms of (x_i, y_i) up to a $U(1)$ transformation. The action of the non-dynamical $U(1)$ is manifestly free on the manifold defined by the equation $\vec{P}_{aux} = 0$, thus, the quotient manifold is regular [18]. Despite the negative kinetic term for $(\tilde{x}, \tilde{y})$, crucial for reasons which will be soon explained, the quotient manifold has a positive definite metric.

Next, we choose the sections:

$$\begin{aligned} X^0 &= -\frac{1}{\sqrt{2}}, \quad X^1 = \frac{i}{\sqrt{2}}, \quad X^{aux} = \frac{1}{\sqrt{2}}\Phi^{aux}, \quad X^a = g\frac{1}{\sqrt{2}}\Phi^a, \\ F_0 &= \frac{i}{\sqrt{2}}z, \quad F_1 = \frac{1}{\sqrt{2}}z, \quad F_{aux} = 0, \quad F_a = -\frac{i}{\sqrt{2}g}g_{ab}\Phi^b. \end{aligned} \tag{45}$$

where $g_{ab} = \mathrm{tr}\,(T_a T_b)$. The indices $0, 1$ label the graviphoton and the hidden $U(1)$, while the

index a labels the physical gauge group [5]. The Kähler potential reads (after reintroducing the powers of M_P for the fields in the physical sector):

$$K = -\log\left(z + \bar{z} - \frac{\Phi^a \bar{\Phi}^b \mathrm{tr}\,(T_a T_b)}{M_P^2}\right). \tag{46}$$

Expanding in power of M_P, the first nontrivial term in Φ gives the standard normalization for the gauge fields and their partners.

We have (using eqs. (21) and (41))

$$X^\Lambda \vec{P}_\Lambda = \frac{\vec{\mathcal{M}}}{M_P b_0} + \sum_i \frac{1}{2\sqrt{2}M_P^3} \mathrm{tr}\,\vec{\sigma} Q_i^\dagger \begin{pmatrix} \Phi_i & \\ & -\Phi_i^T \end{pmatrix} Q_i. \tag{47}$$

Notice that the constant contribution from the hidden sector has been chosen of order $1/M_P$, to insure non-vanishing interference with the physical sector. The auxiliary gauge field and $(\tilde{x}, \tilde{y})$ do not appear in eq. (47), since $\vec{P}_{aux} = 0$, but they will appear in the potential through the terms involving k_{aux}. From now on, we will no longer indicate the powers of M_P. The flat limit corresponds to $|\vec{h}| \gg Q, \Phi$.

Now, expand the potential in eq. (13). We obtain the remarkably simple formula

$$\begin{aligned} V = \quad & g_4^2 \mathrm{tr}\,\left(\left[\Phi, \Phi^\dagger\right]\right)^2 + g_4^2 \vec{P}_a^{(4)} \vec{P}_b^{(4)} (g^{-1})_{ab} + 4 g_1^2 \vec{P}^{(1)} \vec{P}^{(1)} \\ & -2\left|X^\Lambda \vec{P}_\Lambda\right|^2 + 4 h_{uv} k_\Lambda^u k_\Sigma^v X^\Lambda X^\Sigma. \end{aligned} \tag{48}$$

Any dependence on $z + \bar{z}$ has been re-absorbed in a rescaling of Φ and $\mathcal{M}$. The first two terms in (48) were already present in eq. (26), and are in the standard rigid Lagrangian for gauge fields and hypermultiplets. The fourth term contains the missing term in eq. (26), needed to complete the rigid Lagrangian. The rest of the third and fourth terms is the interference between the hidden and the physical sector and, thus, gives the soft breaking terms we are looking for.

Let us collect the expressions for the Killing vectors (cfr. eqs. (18,22))

$$\begin{aligned} SO(4,1)/SO(4) \quad &: \quad X^\Lambda k_\Lambda^u = \delta^{ux}\left(\mathcal{M}^x + \frac{\Phi^{aux}}{\sqrt{2}} h^x\right), \\ \tilde{Q} \quad &: \quad X^\Lambda k_\Lambda^u e_u = -ip\frac{\Phi^{aux}}{\sqrt{2}}\sigma_3 \tilde{Q}, \\ Q_i \quad &: \quad X^\Lambda k_\Lambda^u e_u = -iq_i \frac{\Phi^{aux}}{\sqrt{2}}\sigma_3 Q_i - i\frac{1}{\sqrt{2}}\begin{pmatrix} \Phi & \\ & -\Phi^T \end{pmatrix} Q_i. \end{aligned} \tag{49}$$

We see that the square of the second term in the Killing vector for Q exactly reproduces the missing term in the rigid Lagrangian.

[5] In formulas (45,46) only one factor in the gauge group is indicated. Our theory has, obviously, three coupling constants, g_1,g_4, and the $SU(3)$ gauge coupling.

The interference terms in the scalar potential are:

$$
\begin{aligned}
- & \frac{1}{\sqrt{2}}\sum_i \mathrm{tr}\,\frac{\vec{\mathcal{M}}^*}{b_0}\vec{\sigma}Q_i^\dagger\begin{pmatrix}\Phi & \\ & -\Phi^T\end{pmatrix}Q_i + \Phi^{aux}\left[\frac{\sqrt{2}}{b_0^2}\vec{\mathcal{M}}^*\vec{h} + 2\sum_i q_i\left(x_i^\dagger\Phi^\dagger x_i + y_i^t\Phi^\dagger y_i^*\right) + h.c.\right] \\
+ & |\Phi^{aux}|^2\left[\frac{\vec{h}^2}{b_0^2} + \sum_i 2q_i^2\left(|x_i|^2 + |y_i|^2\right) - 2p^2\left(|\tilde{x}|^2 + |\tilde{y}|^2\right)\right]. \qquad (50)
\end{aligned}
$$

Our aim is to reproduce exactly all the soft breaking terms in formula (31). The first term in (50) can reproduce the tri-linear coupling in formula (31), by correctly orienting the vector $\vec{\mathcal{M}}$. We will choose

$$
\frac{\vec{\mathcal{M}}^*}{b_0}\vec{\sigma} = \begin{pmatrix}\epsilon & B \\ 0 & -\epsilon\end{pmatrix} \qquad (51)
$$

If $\epsilon = 0$ we get exactly and only the desired coupling in (31). So we can identify B with $M = O(V\hat{v}/v)$, the largest scale in our theory. The need for ϵ will be clear soon.

The other terms in formula (31) are generated by the quaternionic quotient. As explained before, the auxiliary field Φ^{aux} in formula (50) can be eliminated using its equation of motion and the result must be evaluated on the submanifold:

$$
\vec{P}^{aux} = \frac{\vec{h}}{b_0} + \sum_i \frac{q_i}{2}\mathrm{tr}\,\vec{\sigma}Q_i^\dagger\sigma_3 Q_i - \frac{p}{2}\mathrm{tr}\,\vec{\sigma}\tilde{Q}^\dagger\sigma_3\tilde{Q} = 0. \qquad (52)
$$

This produce soft breaking terms, among which the mass terms for x_i and y_i, with coefficents determined by p and q_i.

The generation of a positive mass term for Φ is more subtle. In the derivation of formula (48) we assumed that the spontaneous symmetry breaking occurs with zero cosmological constant. This is true for the model discussed in section 3, but it is no longer true after taking the quaternionic quotient, which generates a cosmological constant $M_P^2 E_0$. By expanding the factor e^K in front of the potential in formula (13), we see that we generate the mass term $E_0\Phi^2$. The sign of E_0, as we will see, is determined by the sign of the kinetic term for $\tilde{x}$ and $\tilde{y}$. We choose a negative metric for $\tilde{x}$ and $\tilde{y}$ just to generate a positive mass term for Φ.

Let us derive the explicit expression for the soft breaking terms. By keeping only the relevant terms in the M_P expansion in formula (52), and orienting $\vec{h}$ in the direction 3 ($\vec{h} = (0, 0, h)$), we find

$$
|\tilde{x}|^2 + |\tilde{y}|^2 = \sqrt{\frac{1}{4}\left(\mathrm{tr}\,\vec{\sigma}\tilde{Q}^\dagger\sigma_3\tilde{Q}\right)\left(\mathrm{tr}\,\vec{\sigma}\tilde{Q}^\dagger\sigma_3\tilde{Q}\right)} \approx \frac{h}{pb_0} + \sum_i \frac{q_i}{p}\left(|x_i|^2 - |y_i|^2\right), \qquad (53)
$$

where we have chosen the same sign for h and p.

Substituting this expression back in eq. (50), eliminating the auxiliary field Φ^{aux}, and expanding the resulting expression, we finally get

$$
-\sqrt{2}By^t\Phi x \quad - \quad \sqrt{2}\epsilon^*\sum_i\left[\left(\frac{2q_i h b_0}{h^2 - 2phb_0} + 1\right)\left(x_i^\dagger\Phi x_i\right) + \left(-\frac{2q_i h b_0}{h^2 - 2phb_0} + 1\right)\left(y_i^t\Phi y_i^*\right)\right] + h.c.
$$

$$+ \sum_i \left| \frac{2q_i h b_0 \epsilon}{h^2 - 2phb_0} \right|^2 \left[2\left(1 - \frac{p}{q_i}\right)|x_i|^2 + 2\left(1 + \frac{p}{q_i}\right)|y_i|^2 \right] + E_0 \mathrm{tr}\,\Phi^2. \tag{54}$$

The cosmological constant reads

$$E_0 = -\frac{2|\epsilon|^2 h^2}{(h^2 - 2phb_0)(z + \bar{z})} \tag{55}$$

We can use the three free parameters ϵ, p, q_i to extablish the hierarchy of scales $V \gg \hat{v} \gg v$. The extra parameter h/b_0 can be fixed in order to get a positive cosmological constant and, as a consequence, a positive mass for Φ. A convenient limit is $p \gg h/b_0$, which gives a positive cosmological constant, whose magnitude can be made as small as desired.

We get finally,

$$\left[-\sqrt{2}By\Phi x - \sqrt{2}\epsilon^* \sum_i \left(1 - \frac{q_i}{p}\right)\left(x_i^\dagger \Phi x_i\right) - \sqrt{2}\epsilon^* \sum_i \left(1 + \frac{q_i}{p}\right)\left(y_i^t \Phi y_i^*\right) + h.c. \right] +$$
$$+2|\epsilon|^2 \sum_i \left|\frac{q_i}{p}\right| \left[\left(1 - \frac{q_i}{p}\right)|x_i|^2 + \left(1 + \frac{q_i}{p}\right)|y_i|^2 \right] + \frac{\epsilon^2 h}{b_0 p(z + \bar{z})} \mathrm{tr}\,\Phi^2. \tag{56}$$

As discussed in Section 2, we need a large positive mass term for y_i, of order $O(g_4 V\hat{v}/v)$, the same as the tri-linear coefficient $y_i^t \Phi x_i$, and a negative mass for x_i, of order $O(g_4 V)$. This can be easily achieved by choosing $B, \epsilon = O(g_4 V\hat{v}/v)$, and by tuning $q_i/p = 1 + O(v^2/\hat{v}^2)$. The term $x_i^\dagger \Phi x_i$ is an undesired one; including it in the minimization of Section 2, would completely change the alignments. Fortunately, the condition that this term is suppressed with respect to the good tri-linear term,

$$\left| B y_i^t \Phi x_i \right| \gg \left| \epsilon^* \left(1 - \frac{q_i}{p}\right) x_i^\dagger \Phi x_i \right|, \tag{57}$$

gives the condition $\hat{v}^2 \gg vV$. This condition was not originally present in [6], but can be easily satisfy without interfering with phenomenological constraints on $\hat{v}$ and V, namely, $\hat{v} = O(100\, GeV)$, $V \gg 1\, TeV$.

5 Conclusions and Comments

In this paper, we have presented an N=2 supergravity model where supersymmetry is spontaneous broken in a hidden sector at a scale $\sqrt{MM_P}$, $M \equiv V\hat{v}/v$. The hidden sector communicates only through interactions of gravitational strength with an observable sector. In the flat limit, supersymmetry breaking affects the observable sector through the appearance of soft terms that explicitly break the *rigid* N=2 supersymmetry, and trigger the breakdown of both the standard-model $SU(2) \times U(1)$, and the symmetry between physical fermions and mirrors. This mechanism is the N=2 analog of the hidden-sector supersymmetry breaking in N=1 supergravity [10].

Our soft supersymmetry breaking terms are those of ref. [6] plus some small sub-dominant terms. The most important of them are the tri-linears

$$\left(1 - \frac{q_i}{p}\right) \epsilon^* x_i \Phi_i x_i. \tag{58}$$

The presence of such terms induces small nonzero VEVs for Φ_{ab}, $a, b = 3, 4$, of order $|(1-q/p)\epsilon| = Vv/\hat{v} \ll \hat{v}$. These nonzero VEVs are not dangerous; indeed, they may even be beneficial to the model since they induce small mixing angles between physical fermions and mirrors. As recalled in Section 2, these mixing angles are favored by recent experimental data [5].

The details of our construction involve two ingredients: the choice of a particular realization of the "special Kähler geometry" for the N=2 vector multiplets, and the extensive use of the technique of quaternionic quotients to define an appropriate manifold for the hypermultiplets.

As explained in Sections 3, 4, we define the special geometry of the vector multiplets by giving $n_V + 1$ holomorphic sections such that no prepotential exists. This choice evades an old no-go theorem [19] which forbids spontaneous breaking of N=2 supergravity to N=1 in flat space. This same choice of sections proves useful here, even though in the present case we do not have any argument to show that it is necessary.

The use of quaternionic quotients is a powerful method to find a "custom made" quaternionic space for the hypermultiplets. This technique is particularly useful in the case where the quaternionic manifolds are non-compact. This is the physically interesting one and, luckily, in this case the technique does not run into the snags that mar its application to the construction of compact manifolds (see [18] for details).

A comment about the cosmological constant and the radiative stability of this model is in order. The introduction of an auxiliary Abelian vector multiplet induces a tree-level cosmological constant of order $M_P^2 m_\Phi^2$ (m_Φ is defined in Section 2). This is not a serious problem, since in any case radiative corrections to the cosmological constant are $O(M_P^2 M^2) \gg M_P^2 m_\Phi^2$: N=2 neither helps solving nor worsens the cosmological constant problem. A more serious problem is that the hierarchy of scales introduced in our model may be destabilized by radiative corrections. This is an important problem well worth investigating; the fact that our effective action is defined at the scale $V\hat{v}/v \ll M_{GUT}$ may render this problem less severe.

Finally, let us comment on the uniqueness of our model. We have not proven that our is the unique way of constructing an N=2 supergravity without light fermions; indeed, the message is the opposite: N=2 supergravity in its most general formulation is more flexible a theory than generally supposed, and it can easily account for a realistic particle spectrum. More general models may conceivably be constructed, which have a zero tree-level cosmological constant, or that, more importantly, give rise to an N=2 grand unified model.

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Appendix A: The Quaternionic Manifold

In our paper we assumed that a quaternionic manifold with some special properties exists. In this appendix, we explicitly construct that manifold, using the method of quaternionic quotients introduced in ref. [18].

As we saw in the paper, to construct our N=2 model we need a quaternionic manifold which reduces to $SO(4,1)/SO(4)$ when the scalars of all Higgs, quark, and lepton hypermultiplets are set to zero. On the other hand, the manifold cannot be an $SO(5,n)/SO(4)\times SO(1,n)$ [6], since this coset structure implies a doubling of quark and lepton generations. This result comes about since in $SO(5,n)/SO(4)\times SO(1,n)$ the hypermultiplets belong to *real* representations of the gauge group. Indeed, in this case, the coset representatives can be written as Q^M_μ, where the index μ labels the fundamental of $SO(4)$ and M labels the vectorial of $SO(1,n)$, and the allowed representations of the physical gauge group must be contained in the vectorial of $SO(1,n)$. For instance, each lepton hypermultiplet (physical plus mirror) in the $(1,4,+1/2)\oplus(1,\bar{4},-1/2)$ is paired with another hypermultiplet in the $(1,4,-1/2)\oplus(1,\bar{4},+1/2)$. We want instead a manifold where some coordinates are represented as in eq. (15) without further constraints.

A quaternionic manifold with the desired properties can be constructed as a quaternionic quotient of the space

$$\mathcal{M}\subset USp(2,n)/USp(1)\times USp(1,n). \tag{A.1}$$

This space can be represented by $n+2$ quaternionic homogeneous coordinates $Q_0,Q_1,...,Q_{n+1}$, identified modulo left multiplication by unit quaternions ($Q_i\sim UQ_i$, $U^\dagger U=1$, $i=0,..,n+1$), and subject to the constraints

$$\begin{aligned} -Q_0^\dagger Q_0+\sum_{i=1}^n Q_i^\dagger Q_i - Q_{n+1}^\dagger Q_{n+1} &= -1. \\ -Q_0^\dagger Q_0+\sum_{i=1}^n Q_i^\dagger Q_i &< 0. \end{aligned} \tag{A.2}$$

[6] As explained in the text, our construction requires a manifold with indefinite metric. The metric is positive definite on the subspace defined by equation (52).

Our quaternionic quotient is defined by coupling the fields $Q_0, .., Q_k$, $k < n+1$ to a non-dynamical $SU(2)$ gauge field acting by right multiplication with unit quaternions:

$$Q_i \to Q_i V, \quad i = 0, .., k, \quad V^\dagger V = 1. \tag{A.3}$$

The resulting quaternionic space is $M \equiv M_0/SU(2)$, where M_0 is the algebraic submanifold of $\mathcal{M}$ defined by setting to zero the prepotentials of the non-dynamical $SU(2)$ [18]:

$$M_0 = \{Q \in \mathcal{M} | -Q_0\sigma_I Q_0^\dagger + \sum_{i=1}^k Q_i \sigma_I Q_i^\dagger = 0\}. \tag{A.4}$$

A subtlety arises at this point: the action of SU(2) on M_0 is not free, and the quotient space M has orbifold singularities. This is most easily seen by defining the non-homogeneous coordinates

$$Q_i = Q_0 q_i, \quad q_0 = 1, \quad Q_0^\dagger Q_0 = 1/(1 - \sum_{i=1}^n q_i^\dagger q_i + q_{n+1}^\dagger q_{n+1}). \tag{A.5}$$

The $SU(2)$ action induced on the q_is by eq. (A.3) is

$$q_i \to V^{-1} q_i V, \quad i = 0, .., k, \quad q_i \to V^{-1} q_i, \quad i = k+1, .., n+1. \tag{A.6}$$

On the subspace of M_0 where q_i vanishes for all $i > k$, the center of $SU(2)$, Z_2, acts trivially, while when $q_i \neq 0$ for some $i > k$, the group acts freely. Moreover, it can be shown that the isotropy group of M_0 is always either Z_2 or the identity [18]. This result implies that our space M is a Z_2 orbifold, with a singularity at $q_i = 0$, $i > k$. It is easy to check this statement explicitly using our coordinates q_i. In terms of them, the constraint in eq. (A.5) becomes independent of q_i, $i > k$:

$$-\sigma_I + \sum_{i=1}^k q_i \sigma_I q_i^\dagger = 0. \tag{A.7}$$

The quotient space $M_0/SU(2)$ can be described very explicitly by using the $SU(2)$ gauge invariance to transform one of quaternions q_i, $i = 1, .., k$, into the diagonal form $a + ib\sigma_3$. This "gauge fixing" leaves only the center Z_2 as residual symmetry, since by the constraint eq. (A.7) not all q_i are diagonal. We must still divide the resulting space by Z_2. This means that topologically M is an open subset of

$$[SO(4, k-3)/SO(4) \times SO(k-3)] \times H^{(1,n-k)}/Z_2, \tag{A.8}$$

where Z_2 acts on $H^{(1,n-k)}$ (the quaternionic hyperplane of dimension $n + 1 - k$ and signature $(1, n-k)$) as:

$$q_i \to -q_i, \quad i > k. \tag{A.9}$$

Moreover, since the metric of M near $q_i = 0$, $\forall i > k$, factorises into the metric of $SO(4, k-3)/SO(4) \times SO(k-3)$ times the *flat* metric of $H^{(1,n-k)}$, the singularity at the origin of $H^{(1,n-k)}$ is an orbifold.

Unlike the case of compact spaces studied in [18], here it is trivial to find a smooth manifold associated to M; it is sufficient to remove the identification given in eq. (A.9)! The resulting space, $\tilde{M}$ is the Z_2 covering of M ($M = \tilde{M}/Z_2$), and is manifestly smooth and quaternionic. Obviously, M is not metrically complete, i.e. it is a part of a larger, possibly singular, quaternionic manifold. This is not a problem for us, since only a small neighborhood of the (smooth) point $q_i = 0$, $i = 1, .., n$, is relevant to the flat limit used in our construction.

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